Exercise 61

Find equations of the tangent line and normal line to the curve at the given point.

$$y = (2+x)e^{-x}, \quad (0,2)$$

Solution

The aim is to find the slope of the tangent and normal lines at x = 0. Take the derivative of y.

$$\frac{d}{dx}(y) = \frac{d}{dx}[(2+x)e^{-x}]$$

$$\frac{dy}{dx} = \left[\frac{d}{dx}(2+x)\right]e^{-x} + (2+x)\left[\frac{d}{dx}(e^{-x})\right]$$

$$= (1)e^{-x} + (2+x)\left[(e^{-x}) \cdot \frac{d}{dx}(-x)\right]$$

$$= e^{-x} + (2+x)[(e^{-x} \cdot (-1)]]$$

$$= e^{-x} - (2+x)e^{-x}$$

$$= [1 - (2+x)]e^{-x}$$

$$= (-1-x)e^{-x}$$

$$= -(1+x)e^{-x}$$

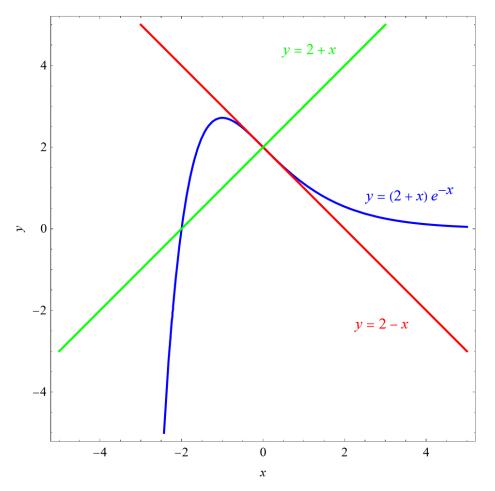
Plug in x = 0 to find the slope of the tangent line at the given point (0, 2). The slope of the normal line is the negative reciprocal.

$$m_{\parallel} = -(1+0)e^{-0} = -1 \quad \Rightarrow \quad m_{\perp} = -\frac{1}{-1} = 1$$

Use the point-slope formula with these slopes and the given point (0, 2) to get the equations of the tangent and normal lines.

y - 2 = 1(x - 0)	y - 2 = -1(x - 0)
y - 2 = x	y - 2 = -x
y = 2 + x	y = 2 - x

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Below is a graph of the curve and its tangent and normal lines at (0, 2).