Exercise 61

Find equations of the tangent line and normal line to the curve at the given point.

$$y = (2 + x)e^{-x}, \quad (0, 2)$$

Solution

The aim is to find the slope of the tangent and normal lines at $x = 0$. Take the derivative of y.

$$\begin{aligned}
\frac{d}{dx}(y) &= \frac{d}{dx}[(2 + x)e^{-x}] \\
\frac{dy}{dx} &= \left[\frac{d}{dx}(2 + x)\right] e^{-x} + (2 + x)\left[\frac{d}{dx}(e^{-x})\right] \\
&= (1)e^{-x} + (2 + x)\left[(e^{-x}) \cdot \frac{d}{dx}(-x)\right] \\
&= e^{-x} + (2 + x)[(e^{-x} \cdot (-1)] \\
&= e^{-x} - (2 + x)e^{-x} \\
&= [1 - (2 + x)]e^{-x} \\
&= (-1 - x)e^{-x} \\
&= -(1 + x)e^{-x}
\end{aligned}$$

Plug in $x = 0$ to find the slope of the tangent line at the given point $(0, 2)$. The slope of the normal line is the negative reciprocal.

$$m_{\parallel} = -(1 + 0)e^{-0} = -1 \quad \Rightarrow \quad m_{\perp} = -\frac{1}{-1} = 1$$

Use the point-slope formula with these slopes and the given point $(0, 2)$ to get the equations of the tangent and normal lines.

$$\begin{aligned}
y - 2 &= -1(x - 0) & \qquad y - 2 &= 1(x - 0) \\
y - 2 &= -x & y - 2 &= x \\
y &= 2 - x & y &= 2 + x
\end{aligned}$$

Below is a graph of the curve and its tangent and normal lines at $(0, 2)$.